# Is Utility Transferable? A Revealed Preference Analysis

Cherchye, L.J.H.; Demuynck, T.; de Rock, B.

*Publication date:*
2011



*Citation for published version (APA):*
Cherchye, L. J. H., Demuynck, T., & de Rock, B. (2011). *Is Utility Transferable? A Revealed Preference Analysis*. (CentER Discussion Paper; Vol. 2011-018). Econometrics.

CentER

Discussion Paper

No. 2011-018

# IS UTILITY TRANSFERABLE? A REVEALED PREFERENCE ANALYSIS

By Laurens Cherchye, Thomas Demuynck and Bram de Rock

10 February 2011

ISSN 0924-7815

TILBURG ◆ UNIVERSITY

# Is utility transferable?
# A revealed preference analysis*

Laurens Cherchye[†], Thomas Demuynck[‡] and Bram De Rock[§]

February 10, 2011

**Abstract**

We provide a revealed preference analysis of the transferable utility hypothesis, which is widely used in economic models. First, we establish revealed preference conditions that must be satisfied for observed group behavior to be consistent with Pareto efficiency under transferable utility. Next, we show that these conditions are easily testable by means of integer programming methods. The tests are entirely nonparametric, which makes them robust with respect to specification errors. Finally, we demonstrate the practical usefulness of our conditions by means of an application to Spanish consumption data. To the best of our knowledge, this is the first empirical test of the transferable utility hypothesis.

**JEL Classification:** C14, DO1, D11, D12, D13.

**Keywords:** transferable utility hypothesis, generalized quasi-linearity, nonparametric tests, revealed preferences.

*We thank Ian Crawford for generously providing us with the data of the Encuesta Continua de Presupestos Familiares (ECPF), which we use in our empirical study.

[†]CentER, Tilburg University and Center for Economic Studies, University of Leuven. E. Sabbelaan 53, B-8500 Kortrijk, Belgium. email: L.Cherchye@uvt.nl and laurens.cherchye@kuleuven-kortrijk.be. Laurens Cherchye gratefully acknowledges financial support from the Research Fund K.U.Leuven through the grant STRT1/08/004.

[‡]Center for Economic Studies, University of Leuven, E. Sabbelaan 53, B-8500 Kortrijk, Belgium. email: thomas.demuynck@kuleuven-kortrijk.be. Thomas Demuynck gratefully acknowledges the Fund for Scientific Research- Flanders (FWO-Vlaanderen) for his postdoctoral fellowship.

[§]ECARES-ECORE, Université Libre de Bruxelles. Avenue F. D. Roosevelt 50, CP 114, B-1050 Brussels, Belgium. E-mail: bderock@ulb.ac.be. Bram De Rock gratefully acknowledges the European Research Council (ERC) for his Starting Grant.

# 1 Introduction

The transferable utility hypothesis plays a crucial role in many economic models. However, the hypothesis is generally conceived as a strong one: it imposes severe restrictions on the underlying utility functions. From this perspective it is somewhat surprising to note that -up till now- the testable implications of transferable utility have hardly received attention in the literature. In fact, to the best of our knowledge, the hypothesis has never been tested on observational data.

This paper fills this gap: we develop tools for investigating the empirical realism of the (theoretically appealing) transferable utility hypothesis. More specifically, we establish revealed preference conditions for observed consumption behavior to be consistent with the transferable utility assumption under Pareto efficient group behavior. These conditions are easily testable as they only require observations on consumed quantities at the group level and corresponding prices; testing the conditions can use standard integer programming methods. In addition, the tests are entirely nonparametric, i.e. their empirical implementation does not require a prior (typically nonverifiable) functional structure for the utility functions of the individuals in the group. We demonstrate the practical usefulness of the conditions by means of an empirical application to a Spanish household consumption data set. This provides a first empirical test of the transferable utility hypothesis.

The remainder of the paper unfolds as follows. In Section 2, we motivate our research question by a brief review of the relevant literature, and we articulate our own contribution. Here we will also indicate that the so-called generalized quasi-linear (GQL) utility specification is the most general specification under which a Pareto optimal allocation rule is consistent with the assumption of transferable utility. In Section 3, we then formally define this GQL specification. Section 4 subsequently presents the corresponding revealed preference characterization. Section 5 provides the integer programming formulation of our characterization and presents the empirical application. Finally, Section 6 concludes.

# 2 Literature review and own contribution

**The popularity of transferable utility.** The transferable utility hypothesis states that it is possible to transfer utility from one person in a group to another person in a lossless manner, i.e. without affecting the aggregate utility of the group. This hypothesis is pervasive in many areas of economics. For example, in cooperative game theory the hypothesis is used to determine the value of a coalition and underlies notions such as the Shapley value (Shapley, 1953), the kernel (Davis and Maschler, 1965) and the nucleolus (Schmeidler, 1969). Next, it is a critical assumption in the Shapley-Shubik assignment model (Shapley and Shubik, 1972), which has become a workhorse in the study of labor and marriage markets and other models of two-sided matching. Furthermore, transferable utility is crucial for the validity of well known theoretical results such as the Coase theorem and Becker's Rotten Kid theorem (see Coase (1960) and Becker (1974); Hurwicz (1995) and Bergstrom (1989) discuss the importance of the transferability hypothesis for these theoretical results). Lastly, the hypothesis is also frequently used in principle agent models, theoretical mechanism design, matching models, public economics, industrial organization, international trade, household economics, and so on.[1]

The transferable utility assumption is popular because it has several highly desirable implications. First of all, it guarantees that group demand behavior displays attractive aggregation properties. In particular, any group of individuals then satisfies the representative agent framework: aggregated demand functions behave as if they were generated by a single individual. Next, the transferable utility hypothesis considerably facilitates welfare analysis. As the distribution of utility over the different members of the group does not influence the final outcome, welfare analysis can focus exclusively on the aggregate utility/welfare. As such, utilizing the transferable utility hypothesis for both theoretical and empirical model building makes life of many economists a lot easier. Nevertheless, despite its wide prevalence in economic models, the empirical implications of transferable utility have hardly been studied.

**Testable (differential) implications of transferable utility.** To define the testable implications of transferable utility at the group level, we need to characterize the underlying utility functions of the individuals within the group. The best-known specification leading to the property of transferable utility is the quasi-linear (QL) utility specification. This specification requires the utility functions of the individuals to be linear in at least one good, usually called the numeraire. Unfortunately, QL utility has strong and unrealistic implications (e.g. absence of income effects for all but a single good, risk neutrality, etc.). In this respect, we also note that QL utility will be strongly rejected in our empirical application that we present below.

In the presence of public goods, Bergstrom and Cornes (1981, 1983) and Bergstrom (1989) showed that a weaker form than QL utility equally implies transferable utility, i.e. so-called 'generalized' quasi-linear (GQL) utility (a term coined by Chiappori (2010)). Interestingly, these authors also showed that this GQL specification is the most general specification that allows for transferable utility. The GQL form can be obtained from the QL specification through multiplication of the numeraire by a function defined in terms of the bundle of public goods. The additional requirement that this function is common to all individuals within the group provides the property of transferable utility.

Recently, Chiappori (2010) derived a set of necessary and sufficient conditions on the aggregate demand function such that it is compatible with a Pareto efficient allocation where group members are endowed with GQL utility functions. As far as we know, this is the first (and –up till now– sole) study that makes the testable implications of transferable utility explicit. In view of our following exposition, we remark that Chiappori adopted a so-called 'differential' approach to characterizing GQL utility: he focused on testable (differential) properties of the group demand function to be consistent with transferable utility. Practical applications of this differential approach then typically require a prior

[1]See, for example, Bergstrom (1997) for an extensive review of applications of the transferable utility hypothesis in household economics.

parametric specification of this demand function, which is to be estimated from the data. As we will indicate below, this implies a most notable difference with the approach that we follow here.

**Revealed preference implications.** We complement Chiappori's findings by establishing testable conditions of transferable utility (or GQL utility) in the revealed preference tradition of Samuelson (1938), Houthakker (1950), Afriat (1967), Diewert (1973) and Varian (1982). In contrast to the differential approach, this revealed preference approach obtains conditions that can be verified by (only) using a finite set of group consumption observations (i.e. prices and quantities) and, thus, it does not require the estimation of a group demand function. As such, a main advantage of these revealed preference conditions is that they are entirely nonparametric: they do not impose any functional form on the utility function (generating a particular group demand function) except for usual regularity conditions.

More specifically, we get necessary and sufficient conditions that enable checking consistency of a given data set with transferable utility. In the spirit of Varian (1982), we refer to this as 'testing' data consistency with transferable utility.[2] As for the practical application of these tests, we will show that our revealed preference conditions can be equivalently reformulated as integer programming constraints. This integer programming formulation allows us to test data consistency with transferable utility by applying standard integer programming solution techniques.[3]

**Further contributions.** At this point, it is worth to indicate two further important differences between our study and the original study of Chiappori (2010), which involve two additional contributions. First, to establish his characterization, Chiappori assumed observability of the numeraire good. However, in practice this numeraire good is typically an 'outside' good, i.e. the amount of money not spent on (observed) consumption, which is usually not observed in real-life applications (including our own application). Given this, our following revealed preference analysis will principally focus on characterizing transferable utility for the case with an unobserved numeraire (or outside good). To obtain this characterization, we will first have to establish the characterization that applies to an observed numeraire.

Another main difference between our study and the one of Chiappori is that we present an empirical application that effectively brings our testable implications to observational data. As indicated above, as far as we know, this provides a first empirical test of the

[2] As is standard in the revealed preference literature, the type of tests that we consider here are 'sharp' tests; either a data set satisfies the data consistency conditions or it does not. In this respect, the literature does suggest methodological extensions to account for measurement error and optimization error in revealed preference tests (see, for example, Varian (1985, 1990)). For compactness, we will not include these (straightforward) extensions in our following analysis.

[3] A similar integer programming approach has been fruitfully applied for revealed preference analysis of multi-person consumption behavior. See, for example, Cherchye, Demuynck, and De Rock (2009, 2011b) and Cherchye, De Rock, and Vermeulen (2011a).

transferable utility hypothesis. Specifically, we verify our revealed preference conditions for a sample of households drawn from the Encuesta Continua De Presupestos Familiares (ECPF), a Spanish consumer expenditure survey. In general, our results are mixed. Although we find the assumption of transferable utility to be realistic for a considerable part of the households under consideration, there is also a substantial share of households whose behavior contradicts the transferable utility assumption. As we will discuss, one possible explanation is that the appropriateness of the transferable utility hypothesis depends on the specific household (characteristics) at hand. Following this interpretation, it may effectively make sense to assume transferable utility for particular categories of households.

As a final remark, we indicate that Brown and Calsamiglia (2007) recently developed a revealed preference characterization of the QL utility specification. By focusing on the GQL utility form, we provide revealed preference conditions for a model that contains this QL specification as a special case. We will explicitly discuss the nested nature of the two utility specifications when introducing our revealed preference characterization; this will show that, for the special case of QL utility, our (GQL) revealed preference conditions reduce to the conditions of Brown and Calsamiglia. In addition, in our empirical exercise we will compare the empirical goodness of the GQL and QL specifications. A main conclusion here will be that many households violate QL utility but are consistent with GQL utility. In other words, at least for this application GQL utility appears to be a considerably more realistic assumption than QL utility.

# 3 Generalized quasi-linear utility

Consider a group with $M$ ($\geq 2$) members. Each member $m$ ($\leq M$) consumes a bundle of $1+N$ private goods $(\mathbf{q}^m, x^m) \in \mathbb{R}_+^{N+1}$ and a bundle of $K$ public goods $\mathbf{Q} \in \mathbb{R}_+^K$. The private good $x^m$ denotes member $m$'s amount of the numeraire. For each $m$, we assume $x^m > 0$ in what follows. In addition, we normalize by setting the price of the numeraire equal to one. Next, the vector $\mathbf{p} \in \mathbb{R}_{++}^N$ represents the normalized price vector for the bundle of private goods $\mathbf{q}^m$, while the vector $\mathbf{P} \in \mathbb{R}_{++}^K$ is the normalized price vector for the bundle of public goods $\mathbf{Q}$.

Utility for member $m$ is represented by the strictly increasing and quasi-concave utility function $u^m(\mathbf{q}^m, x^m, \mathbf{Q})$. The utility functions $u^m$ are said to be of the generalized quasi-linear (GQL) form if there exist a (member-specific) function $b^m : \mathbb{R}_+^{K+N} \to \mathbb{R}$ and a (common) function $a : \mathbb{R}_+^K \to \mathbb{R}_{++}$ such that

$$u^m(\mathbf{q}^m, x^m, \mathbf{Q}) = a(\mathbf{Q})x^m + b^m(\mathbf{Q}, \mathbf{q}^m). \tag{1}$$

Bergstrom and Cornes (1983) have shown that this GQL utility function is the most general functional form that still allows for transferable utility.

The GQL specification encompasses the quasi-linear (QL) specification as a special case. Specifically, if $a(\mathbf{Q}) = a$ for all $\mathbf{Q}$ (i.e. the function value $a(\mathbf{Q})$ is everywhere the same) then the specification in (1) coincides with the QL specification:

$$u^m(\mathbf{q}^m, x^m, \mathbf{Q}) = a\ x^m + b^m(\mathbf{Q}, \mathbf{q}^m). \tag{2}$$

However, if $a(\mathbf{Q})$ varies with the level of public goods, then the GQL specification vastly expands the range of utility functions compatible with transferable utility.

We assume that group decisions are made according to the Pareto criterion: allocations are chosen such that no member can be made better of without reducing the utility of some other group member. In this case, any equilibrium allocation $(\mathbf{q}^1, \ldots, \mathbf{q}^M, x^1, \ldots, x^M, \mathbf{Q})$ minimizes total group expenditures subject to the constraint that every member of the group receives at least some predefined level of utility $\bar{u}^m$. In other words, given a fixed vector of utility levels $(\bar{u}^1, \ldots, \bar{u}^M) \in \mathbb{R}^M_+$, Pareto efficiency imposes that the group decision making process solves the next optimization problem (**OP.1**):

$$\begin{aligned} \min_{(\mathbf{q}^1,\ldots,\mathbf{q}^M,x^1,\ldots,x^M,\mathbf{Q}) \in \mathbb{R}_+^{M(N+1)+K}} & \sum_{m=1}^{M} x^m + \sum_{m=1}^{M} \mathbf{p}\mathbf{q}^m + \mathbf{P}\mathbf{Q} \\ \text{s.t. } & a(\mathbf{Q})x^m + b^m(\mathbf{Q}, \mathbf{q}^m) \geq \bar{u}^m \ (\forall m \leq M). \end{aligned}$$

In view of our following analysis, we develop an equivalent formulation of **OP.1**. To obtain the formulation, we first observe that each constraint will be binding in the solution of **OP.1** because the utility functions $u^m$ are strictly increasing. Using this, and because $x^m > 0$ for all $m$, we can substitute the restrictions in the objective function. As a result, we can equivalently reformulate the original optimization problem as follows (**OP.2**):

$$\min_{(\mathbf{q}^1,\ldots,\mathbf{q}^M,\mathbf{Q}) \in \mathbb{R}_+^{MN+K}} \alpha(\mathbf{Q}) \sum_{m=1}^{M} \bar{u}^m - \sum_{m=1}^{M} \beta^m(\mathbf{q}^m, \mathbf{Q}) + \sum_{m=1}^{M} \mathbf{p}\mathbf{q}^m + \mathbf{P}\mathbf{Q}$$

with

$$\begin{aligned} \alpha(\mathbf{Q}) &= \frac{1}{a(\mathbf{Q})} \\ \beta^m(\mathbf{q}, \mathbf{Q}) &= \frac{b^m(\mathbf{q}, \mathbf{Q})}{a(\mathbf{Q})} \ (\forall m \leq M). \end{aligned}$$

From this equivalent formulation, it is directly clear that the optimal solution of problem **OP.1** only depends on the total amount of utility $\sum_m^M \bar{u}^m$ but not on the specific distribution of this amount over the different group members. This demonstrates the property of transferable utility under GQL.

Standard first order conditions characterize the (interior) solutions of problem **OP.2** if the function $\alpha$ is convex and the function $\beta^m$ is concave. Bergstrom and Cornes (1983) showed that these requirements are equivalent to the condition that the utility functions $u^m$ are quasi-concave (which we assumed before). Next, it is easy to verify that $\alpha$ is decreasing in $\mathbf{Q}$ while $\beta^m$ is increasing in $\mathbf{q}$. In what follows, we will further assume that the function $\beta^m$ is also increasing in $\mathbf{Q}$; this condition is sufficient for $u^m$ to be increasing in $\mathbf{Q}$.

For an optimal solution $(\mathbf{q}^{1*}, \ldots, \mathbf{q}^{M*}, x^{1*}, \ldots, x^{M*}, \mathbf{Q}^*)$ of problem **OP.2**, the first order conditions are as follows:

$$-\frac{\partial \alpha(\mathbf{Q}^*)}{\partial \mathbf{Q}} \sum_{m=1}^{M} \bar{u}^m + \sum_{m=1}^{M} \frac{\partial \beta^m(\mathbf{q}^{m*}, \mathbf{Q}^*)}{\partial \mathbf{Q}} = \mathbf{P}, \tag{foc.1}$$

$$\frac{\partial \beta^m(\mathbf{q}^{m*}, \mathbf{Q}^*)}{\partial \mathbf{q}^m} = \mathbf{p}, \tag{foc.2}$$

$$\frac{x^{m*}}{\alpha(\mathbf{Q}^*)} + \frac{\beta(\mathbf{q}^{m*}, \mathbf{Q}^*)}{\alpha(\mathbf{Q}^*)} = \bar{u}^m. \tag{foc.3}$$

Conditions (foc.1) and (foc.2) provide a formal expression of the group's marginal decision rules for the public and private goods, respectively. Next, condition (foc.3) complies with the GQL utility specification in (1). The first order conditions (foc.1)-(foc.3) provide a useful starting point for developing our revealed preference characterization in the next section.

# 4 Revealed preference characterization

We will empirically analyze the aggregate consumption behavior of a group with $M$ individuals, by starting from a finite set of $T$ observed group choices. For each observation $t \in T$, we know the privately and publicly consumed quantities $\mathbf{q}_t$ and $\mathbf{Q}_t$, as well as the corresponding prices $\mathbf{p}_t$ and $\mathbf{P}_t$. (Remark that we only observe the aggregate private quantities $\mathbf{q}_t$ and not the member-specific quantities $\mathbf{q}_t^m$.) In a first instance we assume that the aggregate amount of the numeraire ('outside') good at every $t$ (i.e. $x_t$) is also observed. Later on we will relax this assumption. As discussed before, we believe an unobserved numeraire is the more realistic assumption for real life applications.

**Numeraire observed.** If the consumption of the numeraire is observed, we obtain the data set $S = \{\mathbf{p}_t, \mathbf{P}_t; x_t, \mathbf{q}_t, \mathbf{Q}_t\}_{t \in T}$. In what follows, we present necessary and sufficient conditions for the set $S$ to be rationalizable in terms of GQL utility functions, i.e. there exist functions $\alpha$ and $\beta^m$ so that each bundle $(x_t, \mathbf{q}_t, \mathbf{Q}_t)$ $(t \in T)$ leads to a solution for **OP.2**. This provides a characterization of transferable utility in the revealed preference tradition. Our starting definition is the following:

**Definition 1 (TU-rationalizable)** *The data set $S = \{\mathbf{p}_t, \mathbf{P}_t, x_t, \mathbf{q}_t, \mathbf{Q}_t\}_{t \in T}$ is **transferable utility (TU)-rationalizable** if (i) there exist a convex and decreasing function $\alpha : \mathbb{R}_+^K \to \mathbb{R}$ and $M$ concave and increasing functions $\beta^m : \mathbb{R}_+^{N+M} \to \mathbb{R}^N$ and (ii), for each $t$, there exist private consumption bundles $\mathbf{q}_t^1, \ldots, \mathbf{q}_t^M$ that sum to $\mathbf{q}_t$ and strictly positive numbers $x_t^1, \ldots, x_t^M$ that sum to $x_t$ such that $\{\mathbf{q}_t^1, \ldots, \mathbf{q}_t^M, \mathbf{Q}_t\}$ solves **OP.2** given the prices $\mathbf{p}_t, \mathbf{P}_t$ and utility levels $\bar{u}_t^m = \dfrac{x_t^m}{\alpha(\mathbf{Q}_t)} + \dfrac{\beta^m(\mathbf{q}_t^m, \mathbf{Q}_t)}{\alpha(\mathbf{Q}_t)}$.*

Of course, the above definition could equally well have been stated by using the functions $a$ and $b^m$ and by referring to program **OP.1**. We opt for the current statement to enhance the interpretation of the revealed preference characterization below.

It follows from Definition 1 that the concept of TU-rationalizability implicitly depends on the number $M$ of individuals within the group. However, as the following result shows, this qualification is actually irrelevant in view of practical applications: it is empirically impossible to distinguish between different group sizes; there exists a rationalization of the set $S$ in terms of a single individual (i.e. $M = 1$) if and only if there exists one in terms of any number $M$ of individuals. More specifically, we can prove the following result:[4]

**Proposition 1** *Consider a data set $S = \{\mathbf{p}_t, \mathbf{P}_t, x_t, \mathbf{q}_t, \mathbf{Q}_t\}_{t\in T}$. The following statements are equivalent:*

1. *The data set $S$ is TU-rationalizable for a group of $M$ individuals;*

2. *The data set $S$ is TU-rationalizable for a group of a single individual;*

3. *For all $t \in T$, there exists $\alpha_t, \beta_t, \bar{u}_t \in \mathbb{R}_+$, $\boldsymbol{\lambda}_t^\alpha \in \mathbb{R}_-^K$ and $\boldsymbol{\lambda}_t^\beta \in \mathbb{R}_{++}^K$ such that, for all $t, v \in T$:*

$$\alpha_t - \alpha_v \geq \boldsymbol{\lambda}_v^\alpha(\mathbf{Q}_t - \mathbf{Q}_v), \tag{RP.1}$$

$$\beta_t - \beta_v \leq \mathbf{p}_v(\mathbf{q}_t - \mathbf{q}_v) + \boldsymbol{\lambda}_v^\beta(\mathbf{Q}_t - \mathbf{Q}_v), \tag{RP.2}$$

$$\boldsymbol{\lambda}_t^\beta - \boldsymbol{\lambda}_t^\alpha \bar{u}_t = \mathbf{P}_t, \tag{RP.3}$$

$$\bar{u}_t = \frac{x_t}{\alpha_t} + \frac{\beta_t}{\alpha_t}. \tag{RP.4}$$

The equivalence between statements 1 and 2 demonstrates the aggregation property of the transferable utility assumption that we mentioned above: if a data set is TU-rationalizable for a group of $M$ individuals, then it is rationalizable for a single individual (endowed with a GQL utility function), and vice versa.[5] Statement 3 then provides the combinatorial conditions that characterize the collection of data sets that are TU-rationalizable. The first two conditions ((RP.1) and (RP.2)) define so-called Afriat inequalities (see also our discussion of Afriat's Theorem in Appendix B) that apply to our specific setting; in terms of Definition 1 these inequalities correspond to, respectively, the (convex) function $\alpha$ and the (concave) function $\beta$ (where we drop the index $m$ because of the equivalence between statements 1 and 2). The vectors $\boldsymbol{\lambda}_t^\alpha$ and $\boldsymbol{\lambda}_t^\beta$ then represent the gradient vectors of these functions in terms of the public goods bundle. Finally, the conditions (RP.3) and (RP.4) give the revealed preference counterparts of the first order conditions (foc.1) and (foc.3) that we discussed in the previous section.

**Numeraire unobserved.** In real life applications the amount of the numeraire good is usually not observed. For example, this will also be the case in our own application. The relevant data set is then given as $S = \{\mathbf{p}_t, \mathbf{P}_t; \mathbf{q}_t, \mathbf{Q}_t\}_{t\in T}$.

Interestingly, the result in Proposition 1 enables us to derive a characterization of transferable utility for such a data set $S$. Specifically, we can derive the following result:

[4]Appendix A contains the proofs of our main results.

[5]Chiappori (2010) obtained a similar result in his differential setting.

**Proposition 2** *Consider a data set $S = \{\mathbf{p}_t, \mathbf{P}_t, \mathbf{q}_t, \mathbf{Q}_t\}_{t\in T}$. The following statements are equivalent:*

1. *For each $t \in T$, there exist $x_t \in \mathbb{R}_{++}$ such that $\{\mathbf{p}_t, \mathbf{P}_t, x_t, \mathbf{q}_t, \mathbf{Q}_t\}_{t\in T}$ is TU-rationalizable for a group of $M$ individuals (or, equivalently, a single individual);*

2. *For all $t \in T$, there exist $U_t^A, U_t^B \in \mathbb{R}_+$, $\lambda_t^A \in \mathbb{R}_{++}$, $\tilde{\mathbf{P}}_t^A \in \mathbb{R}_+^K$, $\tilde{\mathbf{P}}_t^B \in \mathbb{R}_{++}^K$ such that, for all $t, v \in T$:*

$$U_t^A - U_v^A \leq \lambda_t^A \left[\tilde{\mathbf{P}}_v^A(\mathbf{Q}_t - \mathbf{Q}_v)\right], \tag{RP.5}$$

$$U_t^B - U_v^B \leq \mathbf{p}_v(\mathbf{q}_t - \mathbf{q}_v) + \tilde{\mathbf{P}}_v^B(\mathbf{Q}_t - \mathbf{Q}_v), \tag{RP.6}$$

$$\tilde{\mathbf{P}}_t^A + \tilde{\mathbf{P}}_t^B = \mathbf{P}_t. \tag{RP.7}$$

When compared to the characterization in Proposition 1, the conditions (RP.5), (RP.6) and (RP.7) in Proposition 2 correspond to (RP.1), (RP.2) and (RP.3), respectively. We refer to the proof of the result for an explicit construction. This proof also shows that, for each observation $t$, we can always construct a numeraire quantity $x_t$ that meets condition (RP.4) if the data satisfy (RP.5)-(RP.7).

One important note pertains to the fact that condition (RP.5), which contains Afriat inequalities, is nonlinear in terms of the unknown $\lambda$'s and $\tilde{\mathbf{P}}^A$'s. From a practical point of view, this nonlinearity makes it difficult to empirically verify the characterization in Proposition 2. However, in the next section we will show that it is possible to equivalently reformulate the conditions (RP.5)-(RP.7) as integer programming constraints. As such, we can use standard integer programming solution methods for verifying transferable utility. Our empirical application will demonstrate the practical usefulness of this integer programming approach.

**Special case: QL rationalizability.** As a concluding exercise, we show that our characterization of GQL utility nests the revealed preference characterization of QL utility that was developed by Brown and Calsamiglia (2007, Theorem 2.2). To see this, we recall from the previous section that QL utility imposes that the function value $\alpha(\mathbf{Q})$ is constant for all $\mathbf{Q}$. In terms of the characterization in Proposition 1, this means that the gradient vector $\boldsymbol{\lambda}_t^\alpha$ equals zero. And, thus, (RP.1)-(RP.4) reduce to

$$\beta_t - \beta_v \leq \mathbf{p}_t(\mathbf{q}_t - \mathbf{q}_v) + \mathbf{P}_t(\mathbf{Q}_t - \mathbf{Q}_v), \tag{RP.8}$$

which exactly obtains the revealed preference conditions originally provided by Brown and Calsamiglia. We also observe that the QL condition (RP.8) is independent of the level for the numeraire ($x_t$), which implies a notable difference with our above characterization of GQL utility. In fact, this independence is also revealed by the fact that the conditions (RP.5)-(RP.7) in Proposition 2 equally coincide with (RP.8) if we set $\tilde{\mathbf{P}}_t^A$ equal to zero for all $t \in T$ (which has a similar meaning as $\boldsymbol{\lambda}_t^\alpha = \mathbf{0}$ in Proposition 1).

# 5 Empirical Application

In this section, we will use the above revealed preference characterization to empirically assess the validity of the transferable utility hypothesis for Spanish household data. As discussed in Section 2, the transferable utility hypothesis has often been used in a (theoretical) household context, which -in our opinion- makes this an interesting application area for demonstrating the practical usefulness of our empirical characterization. In our application, a main focus will be on comparing the performance of the GQL and QL utility specifications. As we motivated before, we will concentrate on the case where the quantity of the numeraire good is not observed. Before presenting our data and results, we first introduce the integer programming formulation of the conditions (RP.5)-(RP.7) in Proposition 2.

**Integer programming formulation.** As indicated above, the conditions (RP.6) and (RP.7) in Proposition 2 are linear and hence easily verifiable, while the Afriat inequalities in condition (RP.5) are nonlinear. However, these Afriat inequalities can be equivalently restated in terms of integer programming constraints by making use of the Generalized Axiom of Revealed Preferences (GARP); this follows from Afriat's Theorem that we recapture in Appendix B.

Let us consider a general setting with a set $Z = \{\mathbf{w}_l; \mathbf{x}_l\}_{l \in L}$ containing (strictly positive) price vectors $\mathbf{w}_l$ and (positive) quantity vectors $\mathbf{x}_l$. Then the GARP condition is as follows:

**Definition 2** *Consider a set* $Z = \{\mathbf{w}_l; \mathbf{x}_l\}_{l \in L}$ *with* $\mathbf{w}_l \in \mathbb{R}^K_{++}$ *and* $\mathbf{x}_l \in \mathbb{R}^K_{+}$*. For any* $l_1, l_2 \in L$*,* $\mathbf{x}_{l_1} R \mathbf{x}_{l_2}$ *if* $\mathbf{w}_{l_1}\mathbf{x}_{l_1} \geq \mathbf{w}_{l_1}\mathbf{x}_{l_2}$*. Next,* $\mathbf{x}_{l_1} R \mathbf{x}_{l_2}$ *if there exists a sequence* $r, \ldots, t$ *(with* $r, \ldots, t \in L$*) such that* $\mathbf{x}_{l_1} R \mathbf{x}_r$*,...,*$\mathbf{x}_t R \mathbf{x}_{l_2}$*. The set* $Z$ *satisfies GARP if, for all* $l_1, l_2 \in L$*,* $\mathbf{x}_{l_1} R \mathbf{x}_{l_2}$ *implies* $\mathbf{w}_{l_2}\mathbf{x}_{l_1} \geq \mathbf{w}_{l_2}\mathbf{x}_{l_2}$*. We refer to* $R$ *as a revealed preference relation.*

We then have the following proposition, which makes use of the binary variables $r_{t,v}$.

**Proposition 3** *Consider a data set* $S = \{\mathbf{p}_t, \mathbf{P}_t, \mathbf{q}_t, \mathbf{Q}_t\}_{t \in T}$*. The following statements are equivalent:*

1. *For each* $t \in T$*, there exist* $x_t \in \mathbb{R}_{++}$ *such that* $\{\mathbf{p}_t, \mathbf{P}_t, x_t, \mathbf{q}_t, \mathbf{Q}_t\}_{t \in T}$ *is TU-rationalizable for a group of* $M$ *individuals (or, equivalently, a single individual);*

2. *For all* $t, v \in T$*, there exist* $r_{t,v} \in \{0,1\}, U_t^A, U_t^B \in \mathbb{R}_+$*,* $\tilde{\mathbf{P}}_t^A \in \mathbb{R}^K_+$*,* $\tilde{\mathbf{P}}_t^B \in \mathbb{R}^K_{++}$ *such that, for all* $t, v, s \in T$*:*

$$U_t^B - U_v^B \leq \mathbf{p}_v(\mathbf{q}_t - \mathbf{q}_v) + \tilde{\mathbf{P}}_v^B(\mathbf{Q}_t - \mathbf{Q}_v), \tag{IP.1}$$

$$\tilde{\mathbf{P}}_t^A + \tilde{\mathbf{P}}_t^B = \mathbf{P}_t, \tag{IP.2}$$

$$\tilde{\mathbf{P}}_t^A(\mathbf{Q}_t - \mathbf{Q}_v) < r_{t,v}\mathbf{P}_t\mathbf{Q}_t, \tag{IP.3}$$

$$r_{t,v} + r_{v,s} \leq 1 + r_{t,s}, \tag{IP.4}$$

$$\tilde{\mathbf{P}}_t^A(\mathbf{Q}_t - \mathbf{Q}_v) \leq (1 - r_{v,t})\mathbf{P}_t\mathbf{Q}_t. \tag{IP.5}$$

The linear inequalities (IP.1) and (IP.2) are clearly identical to (RP.6) and (RP.7). The nonlinear inequalities (RP.5) are replaced by the linear inequalities (IP.3)-(IP.5) that make use of real and integer variables. More specifically, (IP.3)-(IP.5) correspond to the GARP condition in Definition 2 in which we take $\mathbf{w}_l = \tilde{\mathbf{P}}_l^A$ and $\mathbf{x}_l = \mathbf{Q}_l$.

To explain the inequalities (IP.3)-(IP.5), we interpret the variables $r_{t,v}$ in terms of the revealed preference relation $R$, i.e. $r_{t,v} = 1$ corresponds to $\mathbf{Q}_t R \mathbf{Q}_v$. The constraint (IP.3) then imposes $\mathbf{Q}_t R \mathbf{Q}_v$ (or $r_{t,v} = 1$) whenever $\tilde{\mathbf{P}}_t^A \mathbf{Q}_t \geq \tilde{\mathbf{P}}_t^A \mathbf{Q}_v$. Next, the constraint (IP.4) complies with transitivity of the relation $R$: if $\mathbf{Q}_t R \mathbf{Q}_v$ ($r_{t,v} = 1$) and $\mathbf{Q}_v R \mathbf{Q}_s$ ($r_{v,s} = 1$), then $\mathbf{Q}_t R \mathbf{Q}_s$ ($r_{t,s} = 1$). Finally, the constraint (IP.5) states that, if $\mathbf{Q}_v R \mathbf{Q}_t$ ($r_{v,t} = 1$), then we must have $\tilde{\mathbf{P}}_t^A \mathbf{Q}_t \leq \tilde{\mathbf{P}}_t^A \mathbf{Q}_v$.

For a given data set $S$, we can verify the above linear inequalities by using mixed integer linear programming techniques. Given the result in Proposition 3, this effectively checks whether the set $S$ is consistent with transferable utility (i.e. rationalizable in terms of GQL utility functions).

**Application set-up.** Our data are drawn from the Encuesta Continua de Presupestos Familiares (ECPF). The ECPF is a quarterly budget survey (1985–1997) that interviews about 3200 Spanish households on their consumption expenditures. For each household, the data provides consumption observations for a maximum of eight consecutive quarters. See Browning and Collado (2001) and Crawford (2010) for a more detailed explanation of this data set.

For obvious reasons, we focus on households with at least two household members. Next, all households in our sample are headed by a married couple where the husband is full time employed and the wife is outside the labor force. Finally, we exclude all households with less then eight observations. In the end, this obtains a panel with 1585 households.

For each household, we have consumption data (quantities and prices) for 15 nondurable consumption goods: (i) food and non-alcoholic drinks at home; (ii) alcohol; (iii) tobacco; (iv) energy at home (heating by electricity); (v) services at home (heating: not electricity, water, furniture repair); (vi) nondurables at home (cleaning products); (vii) non-durable medicines; (viii) medical services; (ix) transportation; (x) petrol; (xi) leisure (cinema, theatre, clubs for sports); (xii) personal services; (xiii) personal nondurables (toothpaste, soap); (xiv) restaurants and bars and (xv) traveling (holiday). We will treat energy at home, services at home and nondurables at home as our three public goods. To obtain normalized prices, we deflate the price (index) for each good (category) by the value of the consumer price index in the corresponding quarter.

To avoid (debatable) preference homogeneity assumptions across similar households, we will consider each household separately in our following analysis. In other words, we consider a different data set $S = \{\mathbf{p}_t, \mathbf{P}_t; \mathbf{q}_t, \mathbf{Q}_t\}_{t \in T}$ for every individual household. This practice effectively accounts for inter-household heterogeneity and, thus, optimally exploits the panel structure of our data set.

As mentioned above, we will mainly concentrate on comparing the empirical validity of the GQL and QL utility specifications, of which the rationalizability conditions have been given in Section 4. Next, as a further base of comparison, we will also check for each household-specific data set $S$ whether it satisfies GARP. As explained more formally in Appendix B, such GARP consistency is equivalent to rationalizability in terms of some (possibly non-(G)QL) utility function. Because the GARP condition is most frequently considered in empirical applications of revealed preference analysis, we feel this provides a useful benchmark for evaluating the (G)QL specifications that form the central focus of our analysis. In fact, this GARP test can be given a specific interpretation in our setting: because we do not explicitly use information on the numeraire good, consistency of $S$ with GARP effectively constitutes a necessary and sufficient condition for rationalizability in terms of a utility function $U$ that is separable in the bundle $(\mathbf{q}, \mathbf{Q})$ and the numeraire quantity $x$, i.e. $U(\mathbf{q}, \mathbf{Q}, x) = U(v(\mathbf{q}, \mathbf{Q}), x)$.[6]

**Empirical results: pass rates and power.** In what follows, we do not only consider the mere test results but also the discriminatory power of the three (GQL, QL and GARP) rationalizability conditions under study. Indeed, Bronars (1987) and, more recently, Andreoni and Harbaugh (2008) and Beatty and Crawford (2011) -rather convincingly- argue that revealed preference test results (indicating pass or fail of the data for some behavioral condition) should be complemented with power measures to obtain a fair empirical assessment of the condition under evaluation. Indeed, favorable test results (i.e. a high pass rate for some given data), which prima facie suggest a good empirical fit, have little value if the test has little discriminatory power (i.e. the condition is hard to reject for the data at hand).

For each of the three rationalizability conditions under evaluation, we compute a power measure for every individual household. This measure quantifies discriminatory power in terms of the probability to detect random behavior, and is constructed as follows. We model random behavior by using a bootstrap procedure: we simulate 1000 random series of eight consumption choices by constructing, for each of the eight observed household budgets, a random quantity bundle exhausting the given budget (for the corresponding prices). We construct these quantity bundles by randomly drawing (with replacement) budget shares (for the 15 goods) from the set of 12680 (= 8 x 1585) observed household choices in our data set. The power measure is then calculated as one minus the proportion of these randomly generated consumption series that are consistent with the rationalizability condition under evaluation. By using this bootstrap method, our power assessment gives information on the expected distribution of violations under random choice, while incorporating information on the households' actual choices.[7]

Table 1 presents our results. The first column in the table gives the pass rates for the

[6]Observe that this is not equivalent to testing rationalizability in terms of a general (possibly non-separable) utility function $U$. Indeed, as shown by Varian (1988), as soon as we do not observe the consumption quantity of some good (in casu the numeraire quantity $x_t$), then there always exists some utility function that rationalizes observed behavior (i.e. we can always construct $x_t$ with price equal to one) such that the resulting set $\{\mathbf{p}_t, \mathbf{P}_t; x_t, \mathbf{q}_t, \mathbf{Q}_t\}_{t \in T}$ satisfies GARP).

[7]We refer to Bronars (1987) and Andreoni and Harbaugh (2008) for a general discussion on alternative procedures to evaluate power in the context of revealed preference tests such as ours.

Table 1: Pass rates and power

| condition | pass rate | power | | | | | |
|---|---|---|---|---|---|---|---|
| | | mean | min | 1st quartile | median | 3rd quartile | max |
| GARP | 0.903 | 0.115 | 0.000 | 0.008 | 0.066 | 0.210 | 0.628 |
| QL | 0 | 1 | 0.998 | 1 | 1 | 1 | 1 |
| GQL | 0.451 | 0.4121 | 0.107 | 0.264 | 0.465 | 0.531 | 0.673 |

three rationalizability conditions. Each pass rate gives the proportion of households in our sample that meets a particular (GQL, QL or GARP) condition. A first observation is that the GARP condition provides a very good fit of the data: more than 90% of the households pass GARP, which means that their behavior can be rationalized by a separable utility function as defined above. Not surprisingly, the stringent GQL condition does worse. Still, we find that about half of all households (i.e. 45%) are consistent with the GQL specification. For these households, we cannot reject the transferable utility hypothesis. Finally, the QL utility specification appears to be overly stringent for the current data set: not a single household passes the corresponding rationalizability condition.

The remaining columns of Table 1 report on the power distribution for the three rationalizability tests. First, for the QL specification we obtain that the power distribution is almost entirely centered around unity, which reveals a (nearly) 100% probability of rejecting random behavior for each individual household. At this point, we note that this high power should not be too surprising given our previous finding that the QL condition is rejected for every household in our sample. Next, if we compare the power distributions for the GQL and GARP conditions, we observe that the discriminatory power is rather substantially higher for the GQL test than for the GARP test. Figure 1 provides corresponding kernel estimations of the GARP and GQL power distributions. A notable observation is that the distribution for the GQL setting is bimodal with peaks around 0.15 and 0.5. Overall, Figure 1 confirms the general picture described before (based on Table 1), i.e. the GQL test is considerably more powerful than the GARP test.

As an additional investigation, we consider two additional power distributions for the GARP and GQL tests. Specifically, for each test we compare the power distribution for the group of households that pass the test (pass group) with the one for the group of households that fail the test (fail group). Table 2 gives the results. As a first observation, we find an obvious trade-off between power and pass rate for the GARP test: the power distribution for the fail group clearly dominates the distribution for the pass group. This suggests that a household passing the GARP test is generally characterized by a lower power for this test than a household that fails the GARP test. Interestingly, this trade-off seems to be less prevalent for the GQL test: in this case, the difference between the power distributions for the pass and fail groups is far less pronounced.

Figure 1: Power

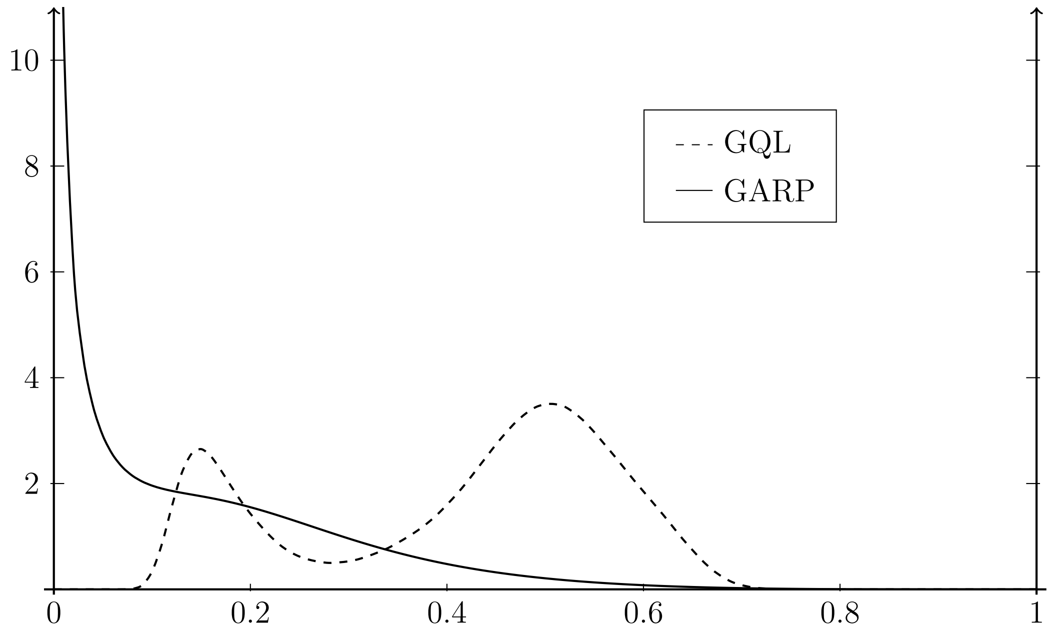



Table 2: power for pass and fail groups

| condition | group | power | | | | | |
|---|---|---|---|---|---|---|---|
| | | mean | min | 1st quartile | median | 3rd quartile | max |
| GARP | pass | 0.102 | 0 | 0.06 | 0.048 | 0.186 | 0.628 |
| | fail | 0.241 | 0.002 | 0.156 | 0.225 | 0.318 | 0.617 |
| GQL | pass | 0.364 | 0.107 | 0.178 | 0.406 | 0.511 | 0.656 |
| | fail | 0.451 | 0.109 | 0.404 | 0.489 | 0.549 | 0.673 |

Table 3: Predictive success

| condition | mean | min | 1st quartile | median | 3rd quartile | max |
|---|---|---|---|---|---|---|
| GARP | 0.018 | -0.998 | 0.001 | 0.034 | 0.163 | 0.628 |
| QL | 0 | -0.002 | 0 | 0 | 0 | 0 |
| GQL | -0.136 | -0.891 | -0.523 | -0.396 | 0.375 | 0.656 |

**Empirical results: predictive success.** As a final exercise, we compute a predictive success measure for the three conditions that we study, which was recently introduced by Beatty and Crawford (2011) and is based on an original proposal of Selten (1991). The measure combines the pass rate and power of a particular behavioral condition into a single metric: for each household, it subtracts 1 minus the power measure from the pass measure (1 or 0). As such, the measure is always situated between -1 and 1. Generally, a higher predictive success value then reveals a better empirical performance of the behavioral condition that is subject to testing. More specifically, a predictive success value that is close to -1 pertains to a household that fails the rationalizability condition (i.e. pass measure equals 0) even though the power of the test is low (i.e. close to 0). Conversely, a predictive success value close to 1 indicates a household that passes the condition (i.e. pass measure equals 1) in a situation where this condition has high power (i.e. close to 1). Finally, a predictive success value that equals exactly zero means that the condition is not informative for the household at hand: the condition does not perform any better than the (uninformative) assumption that households exhibit random consumption behavior (for which the power is 0 and the pass measure equals 1, by construction). For a given household, we will use this zero value as a natural threshold value to identify a rationalizability condition as a 'bad' condition (if predictive success is below zero) or a 'good' condition (if predictive success is above zero).

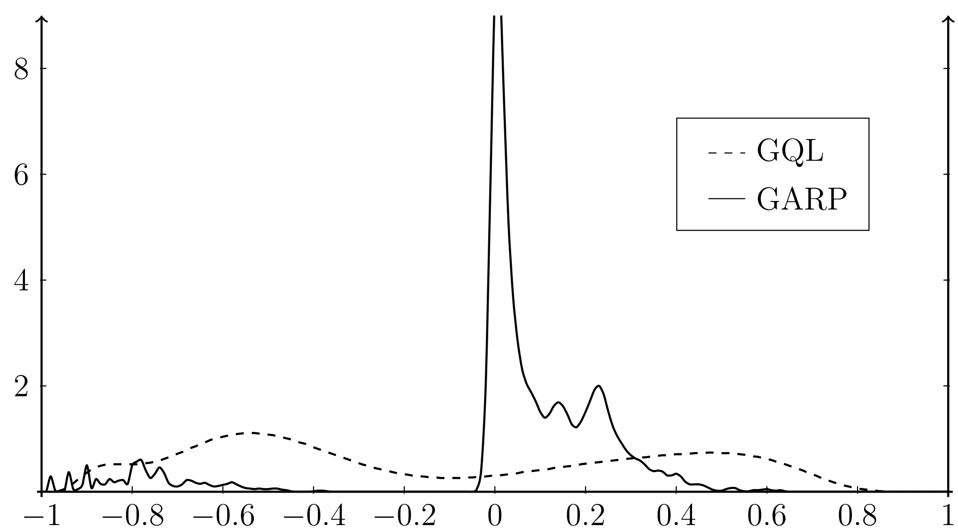
Table 3 gives summary statistics for the predictive success measures that are relevant for our exercise. These statistics tell us about the empirical performance of the three rationalizability conditions at the aggregate level of our sample (with 1585 households). As a first observation, we note that the distribution is centered around zero for the QL condition, with (almost) no variation across observations. In fact, we could have expected this result on the basis of the 0% pass rate and (nearly) 100% power results that we presented before. Given the above, this suggests that the QL condition is not informative for the data at hand. By contrast, the pattern seems to be more indicative for the GARP and GQL conditions. Like for the QL condition, we again get that the mean predictive success score is close to zero, but now there is more variation across households.

To provide a better view of this cross-household variation, Figure 2 depicts estimations of the predictive success distributions for the GQL and GARP conditions. Interestingly, the figure reveals very different pictures for the two conditions. First, for the GARP condition the distribution is largely centered around zero, with a rather limited variation. This suggests that predictive success is (close to) zero for many households in our sample. Like before, we interpret that for these households the GARP condition is not really informative.

However, for a few households the data do allow us to identify GARP as a 'good' or 'bad' behavioral condition (i.e. predictive success is substantially different from zero).

Next, for the GQL condition the predictive success distribution exhibits a clear bimodal pattern: it achieves a first peak around -0.55 and a second peak around 0.5. In our opinion, this bimodality suggests a particular split-up of our original sample of households: for a substantial group of households the GQL condition can be identified empirically as a 'good' one, whereas it is a rather 'bad' condition for the remaining households. In turn, this indicates that the adequacy of the GQL condition may depend on the specific household (characteristics) at hand. In this respect, we have compared observable characteristics (in our data set) for two household groups, i.e. households with predictive success above zero (for which GQL is a 'good' condition) and households with predictive success below zero (for which GQL is a 'bad' condition). We considered the following characteristics: age of the household head, number of household members, specialized worker occupation and home ownership. However, we found no statistical differences between the two subsamples. Therefore, we tentatively conclude that other (unobserved) household characteristics drive the adequacy of the GQL condition, but further research is needed here.

Figure 2: Predictive success




What do we learn from all this? A first conclusion is that, from an empirical point of view, the GQL specification is more useful than the QL specification, which is strongly rejected for our data. In fact, for a considerable subset of households the GQL specification performs rather well empirically; the adequacy of the specification may depend on specific household characteristics (but our data did not allow us to identify which characteristics). For these households, the GQL condition may help to obtain an analysis that is substantially more powerful than the more standard GARP-based analysis. Because the GQL utility specification is the most general specification that is consistent with the assumption of transferable utility, our findings also shed light on the empirical validity of this (theoretically attractive) transferable utility hypothesis. It directly follows that this may effectively constitute a good hypothesis for particular classes of households (but it may also be a rather bad one for other households). At a more general level, we believe that our application convincingly shows the practical usefulness of our revealed preference characterization for assessing the validity of transferable utility (or GQL utility) in real life settings.

# 6 Conclusion

We have presented revealed preference conditions that must be satisfied by observed behavior to be consistent with transferable utility (or GQL utility) under Pareto efficiency. These conditions are easily verified by using mixed integer linear programming techniques, which is attractive from a practical point of view. This provides an easy-to-apply framework for evaluating the empirical realism of the transferable utility hypothesis in real life settings.

We have demonstrated the usefulness of our revealed preference framework by an empirical application to Spanish household data. Our results suggest that the assumption of transferable utility is a useful one for a large class of households in our sample. However, the opposite conclusion applies to other households. Therefore, we tentatively conclude that the adequacy of the hypothesis depends on the specific household (characteristics) at hand.

We see different avenues for follow-up research. First, from an empirical point of view, one may use our framework to more thoroughly investigate the specific household characteristics that drive the adequacy of the transferable utility model (e.g. by using data sets with richer information than the one we studied). We believe this is particularly interesting given the wide use of the transferable utility hypothesis in (theoretical) household economics. Next, referring to our discussion in Section 2, our framework can be used for assessing the validity of the transferable utility hypothesis in alternative (non-household) settings where this assumption crucially underlies important theoretical results.

Finally, to keep our exposition simple, our analysis has concentrated on the characterization of transferable utility, and testing consistency of observed behavior with this characterization. If observed behavior is found consistent with a behavioral hypothesis, then natural next questions involve recovering/identifying the corresponding decision model that rationalizes the observed consumption behavior, and to forecasting behavior in new situations. In this respect, it is worth emphasizing that our revealed preference characterization does allow for subsequent recovery and forecasting analysis. For example, this analysis can develop along the lines of Varian (1982) and, more recently, Blundell, Browning, and Crawford (2008), who considered such questions in a formally similar revealed preference setting. In this respect, we recall from our application that the GQL (or transferable utility) test turns out to be substantially more powerful than the GARP test, which is usually considered in revealed preference applications. As such, we can expect that using the GQL specification (when it cannot be rejected) can effectively produce more vigorous recovery and forecasting results.

# References


Afriat, S. N., 1967. The construction of utility functions from expenditure data. International Economic Review 8, 67–77.

Andreoni, J., Harbaugh, W., 2008. Power indices for revealed preference tests. Tech. rep.

Beatty, T. K. M., Crawford, I. A., 2011. How demanding is the revealed preference approach to demand. American Economic Review forthcoming.

Becker, G. S., 1974. A theory of social interactions. Journal of Political Economy 82, 1063–1093.

Bergstrom, T., 1989. A fresh look at the rotten-kid theorem and other household mysteries. Journal of Political Economy 97, 1138–1159.

Bergstrom, T., 1997. The Handbook of Population and Family Economics. Ch. A Survey of Theories of the Family, pp. 22–74.

Bergstrom, T. C., Cornes, R. C., 1981. Gorman and Musgrave are dual: an antipodean theorem on public goods. Economics Letters 7, 371–378.

Bergstrom, T. C., Cornes, R. C., 1983. Independence of allocative efficiency from distribution in the theory of public goods. Econometrica 51, 1753–1765.

Blundell, R., Browning, M., Crawford, I., 2008. Best nonparametric bounds on demand responses. Econometrica 76, 1227–1262.

Bronars, S. G., 1987. The power of nonparametric tests of preference maximization. Econometrica 55, 693–698.

Brown, D. J., Calsamiglia, C., 2007. The nonparametric approach to applied welfare analysis. Economic Theory 31, 183–188.

Browning, M., Collado, M. D., 2001. The response of expenditures to anticipated income changes: Panel data estimates. American Economic Review 91, 681–692.

Cherchye, L., De Rock, B., Vermeulen, F., 2011a. The revealed preference approach to collective consumption behavior: Testing and sharing rule recovery. Review of Economic Studies 78, 176–198.

Cherchye, L., Demuynck, T., De Rock, B., 2009. Testable implications of general equilibrium models: an integer programming approach. Center for Economic Studies - Discussion papers ces09.14.

Cherchye, L., Demuynck, T., De Rock, B., 2011b. Revealed preference analysis of noncooperative household consumption. The Economic Journal forthcoming.

Chiappori, P.-A., 2010. Testable implications of transferable utility. Journal of Economic Theory 145, 1302–1317.

Coase, R. H., 1960. The problem of social cost. Journal of Law and Economics 3, 1–44.

Crawford, I., 2010. Habits revealed. Review of Economic Studies 77, 1382–1402.

Davis, M., Maschler, M., 1965. The kernel of a cooperative game. Naval Research Logistics Quarterly 12, 223–259.

Diewert, W. E., 1973. Afriat and revealed preference theory. The Review of Economic Studies 40, 419–425.

Houthakker, H. S., 1950. Revealed preference and the utility function. Economica 17, 159–174.

Hurwicz, L., 1995. What is the coase theorem? Japan and the World Economy 7, 49–74.

Samuelson, P. A., 1938. A note on the pure theory of consumer's behavior. Economica 5, 61–71.

Schmeidler, D., 1969. The nucleolus of a characteristic function game. SIAm Journal on Applied Mathematics 17, 1163–1170.

Selten, R., 1991. Properties of a measure of predictive success. Mathematical Social Sciences 21, 153–167.

Shapley, L., Shubik, M., 1972. The assignment game I: The core. International Journal of Game Theory 1, 111–130.

Shapley, L. S., 1953. Contributions to the Theory of Games II. Princeton University Press, Ch. A Value for n-Person Games, pp. 307–317.

Varian, H., 1982. The nonparametric approach to demand analysis. Econometrica 4, 945–974.

Varian, H., 1985. Non-parametric analysis of optimizing behavior with measurement error. Journal of Econometrics 30, 445–458.

Varian, H., 1988. Revealed preference with a subset of goods. Journal of Economic Theory 46, 179–185.

Varian, H., 1990. Goodness-of-fit in optimizing models. Journal of Econometrics 46, 125–140.

# Appendix A: proofs

## Proof of Proposition 1

$(2 \to 3)$. By convexity of the function $\alpha(\mathbf{Q})$ and concavity of the function $\beta(\mathbf{q}, \mathbf{Q})$ we must have that for all observations $t, v \in T$:

$$\alpha(\mathbf{Q}_t) - \alpha(\mathbf{Q}_v) \geq \frac{\partial \alpha(\mathbf{Q}_v)}{\partial \mathbf{Q}} (\mathbf{Q}_t - \mathbf{Q}_v),$$

$$\beta^m(\mathbf{q}_t, \mathbf{Q}_t) - \beta^m(\mathbf{q}_v, \mathbf{Q}_v) \leq \frac{\partial \beta(\mathbf{q}_v, \mathbf{Q}_v)}{\partial \mathbf{q}} (\mathbf{q}_t - \mathbf{q}_v) + \frac{\partial \beta(\mathbf{q}_v, \mathbf{Q}_v)}{\partial \mathbf{Q}} (\mathbf{Q}_t - \mathbf{Q}_v).$$

For all $t \in T$, define $\alpha_t = \alpha(\mathbf{Q}_t)$, $\beta_t = \beta(\mathbf{q}_t, \mathbf{Q}_t)$, $\bar{u}_t = u(x_t, \mathbf{q}_t, \mathbf{Q}_t)$, $\boldsymbol{\lambda}_t^\alpha = \dfrac{\partial \alpha(\mathbf{Q}_t)}{\partial \mathbf{Q}}$ and $\boldsymbol{\lambda}_t^\beta = \dfrac{\partial \beta(\mathbf{q}_t, \mathbf{Q}_t)}{\partial \mathbf{Q}}$. Then, substituting and using the first order conditions (foc.1)-(foc.3) demonstrates conditions (RP.1)-(RP.4).

$(1 \to 3)$ The proof is similar to the case $( 2 \to 3)$ except now, we define $\beta_t = \sum_m \beta^m(\mathbf{q}_t^m, \mathbf{Q}_t)$ and $\boldsymbol{\lambda}_t^\beta = \sum \dfrac{\partial \beta(\mathbf{q}_t^m, \mathbf{Q}_t)}{\partial \mathbf{Q}}$.

$(3 \to 2)$. Define the functions $\alpha(\mathbf{Q})$ and $\beta(\mathbf{q}, \mathbf{Q})$ in the following way:

$$\alpha(\mathbf{Q}) = \max_{t \in T} \left\{ \alpha_t + \boldsymbol{\lambda}_t^\alpha (\mathbf{Q} - \mathbf{Q}_t) \right\}, \tag{A.1}$$

$$\beta(\mathbf{q}, \mathbf{Q}) = \min_{t \in T} \left\{ \beta_t + \mathbf{p}_t(\mathbf{q} - \mathbf{q}_t) + \boldsymbol{\lambda}_t^\beta (\mathbf{Q} - \mathbf{Q}_t) \right\}. \tag{A.2}$$

Define $u(x, \mathbf{q}, \mathbf{Q}) = \dfrac{x}{\alpha(\mathbf{Q})} + \dfrac{\beta(\mathbf{q}, \mathbf{Q})}{\alpha(\mathbf{Q})}$.

The function $\alpha$ is convex and $\beta$ is concave, hence $u$ is quasi-concave. Further, it is increasing in both $\mathbf{q}$ and $\mathbf{Q}$. Finally, using a similar argument as Varian (1982, p.970), we can derive that $\alpha(\mathbf{Q}_t) = \alpha_t$ and $\beta(\mathbf{q}_t, \mathbf{Q}_t) = \beta_t$ for all $t \in T$

Given all this, we can prove the result ad absurdum. Suppose that $S$ is not TU-rationalizable. Then, there must exist an allocation $\{x, \mathbf{q}, \mathbf{Q}\}$ such that $x + \mathbf{p}_t\mathbf{q} + \mathbf{P}_t\mathbf{Q} < x_t + \mathbf{p}_t\mathbf{q}_t + \mathbf{P}_t\mathbf{Q}_t$ and $u(x, \mathbf{q}, \mathbf{Q}) \geq u(x_t, \mathbf{q}_t, \mathbf{Q}_t) = \bar{u}_t$. We thus get

$$\begin{aligned} x + \mathbf{p}_t\mathbf{q} + \mathbf{P}_t\mathbf{Q} &\geq \bar{u}_t \alpha(\mathbf{Q}) - \beta(\mathbf{q}, \mathbf{Q}) + \mathbf{p}_t\mathbf{q} + \mathbf{P}_t\mathbf{Q} \\ &\geq \bar{u}_t \alpha_t - \beta_t + \left( \boldsymbol{\lambda}_t^\alpha \bar{u}_t - \boldsymbol{\lambda}_t^\beta \right) (\mathbf{Q} - \mathbf{Q}_t) - \mathbf{p}_t(\mathbf{q} - \mathbf{q}_t) + \mathbf{p}_t\mathbf{q} + \mathbf{P}_t\mathbf{Q} \\ &= x_t + \mathbf{p}_t\mathbf{q}_t + \mathbf{P}_t\mathbf{Q}_t, \end{aligned}$$

which gives the wanted contradiction. (The first inequality combines $u(x, \mathbf{q}, \mathbf{Q}) = (x/\alpha(\mathbf{Q}))+ (\beta(\mathbf{q}, \mathbf{Q})/\alpha(\mathbf{Q}))$ with $u(x, \mathbf{q}, \mathbf{Q}) \geq \bar{u}_t$, the second inequality uses (A.1) and (A.2), and the final equality uses (RP.3) and (RP.4).)

$(3 \to 1)$ The argument is similar to one for $(3 \to 2)$, when using the additional definition $\beta^m(\mathbf{q}^m, \mathbf{Q}) = \dfrac{1}{M}\beta(M\mathbf{q}^m, \mathbf{Q})$. Then, for all $t \in T$ and $m \leq M$, we set $\mathbf{q}_t^m = \mathbf{q}_t/M$ and $x_t^m = x_t/M$.

## Proof of Proposition 2

$(1 \to 2)$ Assume that there exist numbers $x_t$ such that $\{\mathbf{p}_t, \mathbf{P}_t, x_t, \mathbf{q}_t, \mathbf{Q}_t\}_{t\in T}$ is TU-rationalizable. Then, it follows from Proposition 1 that there exist positive numbers $\alpha_t$, $\beta_t$ and $\bar{u}_t$, vectors $\boldsymbol{\lambda}_t^\alpha \in \mathbb{R}_-^K$ and $\boldsymbol{\lambda}_t^\beta \in \mathbb{R}_{++}^K$ such that

$$\alpha_t - \alpha_v \geq \boldsymbol{\lambda}_v^\alpha(\mathbf{Q}_t - \mathbf{Q}_v) \tag{RP.1}$$

$$\beta_t - \beta_v \leq \mathbf{p}_v(\mathbf{q}_t - \mathbf{q}_v) + \boldsymbol{\lambda}_v^\beta(\mathbf{Q}_t - \mathbf{Q}_v) \tag{RP.2}$$

$$\boldsymbol{\lambda}_t^\beta - \boldsymbol{\lambda}_t^\alpha \bar{u}_t = \mathbf{P}_t \tag{RP.3}$$

$$\bar{u}_t = \frac{x_t}{\alpha_t} + \frac{\beta_t}{\alpha_t} \tag{RP.4}$$

Setting, for all $t \in T$, $\beta_t = U_t^B$, $\boldsymbol{\lambda}_t^\beta = \tilde{\mathbf{P}}_t^B$ and $\tilde{\mathbf{P}}_t^A = -\boldsymbol{\lambda}_t^A \bar{u}_t$ translates condition (RP.2) and (RP.3) into conditions (RP.6) and (RP.7). So we only need to demonstrate condition (RP.5).

Multiplying (RP.1) by minus one, gives:

$$-\alpha_t - (-\alpha_v) \leq \frac{1}{\bar{u}_t}\tilde{\mathbf{P}}_v^A\left(\mathbf{Q}_t - \mathbf{Q}_v\right)$$

Given this, setting $\lambda_t^A = 1/\bar{u}_t > 0$ and $U_t^A = -\alpha_t - \min_v\{-\alpha_v\} \geq 0$ establishes condition (RP.5).

$(2 \to 1)$ Assume that there exist numbers $U_t^A, U_t^B$ and $\lambda_t^A$, and vectors $\tilde{\mathbf{P}}_v^A$ and $\tilde{\mathbf{P}}_v^B$ such that

$$U_t^A - U_v^A \leq \lambda_t^A \left[\tilde{\mathbf{P}}_v^A(\mathbf{Q}_t - \mathbf{Q}_v)\right] \tag{RP.5}$$

$$U_t^B - U_v^B \leq \mathbf{p}_v(\mathbf{q}_t - \mathbf{q}_v) + \tilde{\mathbf{P}}_v^B(\mathbf{Q}_t - \mathbf{Q}_v) \tag{RP.6}$$

$$\tilde{\mathbf{P}}_t^A + \tilde{\mathbf{P}}_t^B = \mathbf{P}_t \tag{RP.7}$$

First, by setting, for all $t \in T$, $\beta_t = U_t^B$, $\boldsymbol{\lambda}_t^\beta = \tilde{\mathbf{P}}_t^B$, we derive (RP.2).

Next, we define $\bar{u}_t = 1\big/\lambda_t^A$ and $\tilde{\mathbf{P}}_t^A\Big/\bar{u}_t = -\boldsymbol{\lambda}_t^\alpha$. Substitution in condition (RP.7) gives condition (RP.3).

Further, multiplying (RP.5) by minus one gives,

$$-U_t^A - (-U_v^A) \geq \boldsymbol{\lambda}_t^\alpha\left(\mathbf{Q}_t - \mathbf{Q}_v\right) \tag{A.3}$$

As $\bar{u}_t > 0$, there exist a number $\delta > 0$ such that $\bar{u}_t > \delta$ for all $t \in T$. Now, consider a number $z \in \mathbb{R}_{++}$ and define $\alpha_t$ such that (i) $\alpha_t \equiv -U_t^A + z > 0$ $(\forall t \in T)$ and (ii)

$0 < \beta_t/\alpha_t \leq \delta$. These conditions can be guaranteed by taking $z$ large enough. Using this definition of $\alpha_t$ in condition (A.3) above gives condition (RP.1).

Finally, we define $x_t$ such that

$$x_t \equiv \alpha_t \bar{u}_t - \beta_t > 0,$$

which obtains condition (RP.4).

### Proof of Proposition 3

This result uses the equivalence of (RP.5) and GARP; this is stated more formally in Theorem 1 below. Next, in the main text we argued that (IP.3)-(IP.5) do allow for verifying GARP for our setting.

## Appendix B: Afriat's Theorem

In the main text, we make use of Afriat's Theorem. This result was stated by Varian (1982) and is based on the original work of Afriat (1967). It is probably the single most important theorem in the revealed preference literature. To facilitate our exposition in the main text, we briefly recapture the result here. We refer to Varian (1982) for more a more detailed discussion.

Let us consider a general setting with a price-quantity set Z as introduced in Section 5 of the main text. We consider the following rationalizability concept:

**Definition 3 (U-rationalizable)** *The set $Z = \{\mathbf{w}_l; \mathbf{x}_l\}_{l \in L}$, with $\mathbf{w}_l \in \mathbb{R}^K_{++}$ and $\mathbf{x}_l \in \mathbb{R}^K_+$, is **utility (U)-rationalizable** if there exist a non-satiated utility function $u$ such that each quantity bundle $\mathbf{x}_l$ maximizes the function $u$ in the following sense: $\mathbf{x}_l \in \arg\max_{\mathbf{x}} u(\mathbf{x})$ s.t. $\mathbf{w}_l \mathbf{x} \leq \mathbf{w}_l \mathbf{x}_l$.*

We can now state Afriat's Theorem.

**Theorem 1 (Afriat's Theorem)** *Consider a set $Z = \{\mathbf{w}_l; \mathbf{x}_l\}_{l \in L}$ with $\mathbf{w}_l \in \mathbb{R}^K_{++}$ and $\mathbf{x}_l \in \mathbb{R}^K_+$. The following conditions are equivalent:*

1. The set $Z$ is U-rationalizable;

2. The set $Z$ satisfies GARP;

3. For all $l \in L$, there exist $U_l \in \mathbb{R}_+$ and $\lambda_l \in \mathbb{R}_{++}$ such that, for all $l, k \in L$ :

$$U_l - U_k \leq \lambda_k \mathbf{w}_k (\mathbf{x}_l - \mathbf{x}_k);$$

4. There exist a strictly increasing, continuous and concave utility function that provides a rationalization for $Z$.

In Section 5 of our main text, we use two important implications of this result. First, the equivalence between statements 1 (or 4) and 3 implies that a price-quantity set $Z$ is U-rationalizable by some utility function if and only if it is consistent with GARP. Second, the equivalence between statements 2 and 3 means that the set $Z$ is consistent with GARP if and only if it satisfies a number of inequalities defined in the unknowns $U_l$ and $\lambda_l$. These last inequalities are commonly referred to as 'Afriat inequalities' corresponding to the set $Z$. Intuitively, these Afriat inequalities allow us to obtain estimates for the utility levels $(U_t)$ and marginal utilities $(\lambda_t)$ attained at each $l$ whenever the set $Z$ is rationalizable.